Thermophysical property-related comparison criteria for nanofluid heat transfer enhancement in turbulent flow

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Heat transfer enhancement criteria for nanofluids over their base fluids are presented based on three separate considerations: Reynolds number, flow velocity, and pumping power. Analyses presented show that, among the three comparisons, the constant pumping power comparison is the most unambiguous; the constant flow velocity comparison can be quite reasonable under certain conditions but the constant Reynolds number comparison (the most commonly used in the engineering literature for nanofluids) distorts the physical situation, and therefore, should not be used. © *2010 American Institute of Physics*. [doi:10.1063/1.3435487]

Comparing forced convective heat transfer performance among fluids, based on the thermophysical properties of the fluids, has long been of interest to the thermal science and heat transfer research community and remains an essential part of selecting optimal fluids for specific applications. Various principles have been proposed for comparing the heat transfer performance among candidate fluids for any particular application. A figure of merit called the Mouromtseff number Mo was introduced in 1942 by Mouromtseff[1] to represent the effect on the heat transfer coefficient h of the combined thermophysical properties of density ρ, specific heat C_p, dynamic viscosity μ, and thermal conductivity k. A more complicated figure of merit was suggested by Yeh and Chu[2] to further capture the combination effect of the fluid heat storage capacity characterized by the specific heat C_p, the heat removal ability characterized by the heat transfer coefficient h, and the hydraulic performance characterized by the friction-related pumping power P. Both of those comparison principles are based on the presumption that all of the fluids in the comparison have the same flow velocity. Other comparisons,[3] related to nanofluids, included fluid temperature considerations.

In recent years, many studies of nanofluid heat transfer have made comparisons to the base fluid heat transfer. However, in most cases principles incorporating constant flow velocity such as the two noted above were not used. Instead, most nanofluid comparisons have been at the same Reynolds number. This practice in the literature has been causing some confusion regarding the advantage of using a nanofluid instead of the base fluid in heat transfer applications. In the present study, three bases were utilized for comparing heat transfer between nanofluids and their base fluids: constant Reynolds number, constant flow velocity, and constant pumping power.

For a forced, single-phase, turbulent, internal flow with the flow channel diameter of d, the flow channel length of L, and the flow velocity of V, the Nusselt number Nu can be predicted from the Reynolds number Re, the Prandtl number Pr, and the friction factor f of the fluid. From among the commonly-used prediction equations (the Dittus–Boelter equation,[4] the Petuekhov–Popov equation,[5] and the Gnielinski equation[6]) for the turbulent Nusselt number, the Dittus–Boelter equation is chosen in the present study for simplicity. In terms of the heat transfer coefficient, the Dittus–Boelter equation can be expressed as

$$h = \mathrm{Nu}(k/d) = 0.023\ \mathrm{Re}^{4/5}\ \mathrm{Pr}^{2/5}(k/d) = 0.023 d^{-1/5}\rho^{4/5}C_p^{2/5}\mu^{-2/5}k^{3/5}V^{4/5}. \quad (1)$$

Based on the standard Blasius equation[7] for the Fanning friction factor

$$f_{\mathrm{Blasius}} = 0.0791\ \mathrm{Re}^{-1/4} = 0.0791 d^{-1/4}\rho^{-1/4}\mu^{1/4}V^{-1/4}, \quad (2)$$

the friction-related pumping power P can be calculated from the volume flow rate Q and the pressure drop Δp as

$$P = Q\Delta p = (\pi d^2 V/4)(2 f_{\mathrm{Blasius}}\rho V^2 L/d) = 0.03955\pi d^{3/4}L\rho^{3/4}\mu^{1/4}V^{11/4}. \quad (3)$$

To compare the fluid-property-related heat transfer performance of a nanofluid to that of the base fluid on the constant Reynolds number basis, constant flow velocity basis, and constant pumping power basis, the base fluid properties, particle properties, and nanofluid properties are denoted by the subscripts m, p, and e, respectively. A figure of merit $r = h_e/h_m$ for the heat transfer coefficient ratio of the nanofluid over the base fluid is used to compare the heat transfer performance of the nanofluid to that of the base fluid. From the viewpoint of the heat transfer coefficient, a beneficial nanofluid means that the figure of merit r is of a value greater than unity. For the constant Reynolds number comparison with $\rho_e V_e/\mu_e = \rho_m V_m/\mu_m$, the figure of merit r_{Re} is

$$r_{\mathrm{Re}} = \left(\frac{h_e}{h_m}\right)_{\mathrm{Re}} = \left(\frac{C_{pe}}{C_{pm}}\right)^{2/5}\left(\frac{\mu_e}{\mu_m}\right)^{2/5}\left(\frac{k_e}{k_m}\right)^{3/5}. \quad (4)$$

This figure of merit r_{Re} is the most commonly used in the literature for nanofluid heat transfer enhancement, and it implies the following flow velocity ratio of the nanofluid over the base fluid

$$(V_e/V_m)_{\mathrm{Re}} = (\rho_e/\rho_m)^{-1}(\mu_e/\mu_m). \quad (5)$$

In most cases, the flow velocity ratio for the constant Reynolds number comparison is greater than unity, and, as will be discussed subsequently, it is this feature that is misleading in the heat transfer comparison.

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For the constant flow velocity comparison with $V_e = V_m$, the figure of merit r_V is equivalent to the Mouromtseff number[1] ratio and can be expressed as

$$r_V = \left(\frac{h_e}{h_m}\right)_V = \left(\frac{\rho_e}{\rho_m}\right)^{4/5}\left(\frac{C_{pe}}{C_{pm}}\right)^{2/5}\left(\frac{\mu_e}{\mu_m}\right)^{-2/5}\left(\frac{k_e}{k_m}\right)^{3/5}. \quad (6)$$

For the constant friction-related pumping power comparison with $\rho_e^{3/4}\mu_e^{1/4}V_e^{11/4} = \rho_m^{3/4}\mu_m^{1/4}V_m^{11/4}$, the figure of merit r_P is

$$r_P = \left(\frac{h_e}{h_m}\right)_P = \left(\frac{\rho_e}{\rho_m}\right)^{32/55}\left(\frac{C_{pe}}{C_{pm}}\right)^{2/5}\left(\frac{\mu_e}{\mu_m}\right)^{-26/55}\left(\frac{k_e}{k_m}\right)^{3/5}. \quad (7)$$

The flow velocity ratio of the nanofluid over the base fluid associated with the constant pumping power comparison is

$$(V_e/V_m)_P = (\rho_e/\rho_m)^{-3/11}(\mu_e/\mu_m)^{-1/11}. \quad (8)$$

In most cases, the flow velocity ratio for the constant pumping power comparison is only slightly less than unity due to the small exponents $-3/11$ for the density ratio $\rho_e/\rho_m > 1$ and $-1/11$ for the dynamic viscosity ratio $\mu_e/\mu_m > 1$.

From the relations that $r_{\mathrm{Re}}/r_V = (\rho_e/\rho_m)^{-4/5}(\mu_e/\mu_m)^{4/5}$ and $r_V/r_P = (\rho_e/\rho_m)^{12/55}(\mu_e/\mu_m)^{4/55}$, the following inequality is generally true $r_{\mathrm{Re}} > r_V > r_P$ although, as will be shown, r_V is often approximate to r_P for nanofluids.

It can be seen from the above derivation that the essential factor, which separates the three comparison bases from each other, is the flow velocity ratio V_e/V_m depending on the density ratio ρ_e/ρ_m and the dynamic viscosity ratio μ_e/μ_m. For nanofluids with a typical particle volume concentration of $<5\%$, the density ratio ρ_e/ρ_m and the specific heat ratio C_{pe}/C_{pm} are close to unity. (The later minimizes errors due to the implicit assumption in the figure of merit r that the wall-to-fluid temperature difference is the same for nanofluid and base fluid.) Therefore, the flow velocity ratio V_e/V_m is mainly determined by the dynamic viscosity ratio μ_e/μ_m. To clearly show the influence of the flow velocity ratio V_e/V_m on the heat transfer coefficient ratio h_e/h_m, these parameters are plotted in Fig. 1 for the three comparison bases as a function of the dynamic viscosity ratio μ_e/μ_m for a SiC-50/50 ethylene glycol/water nanofluid with a particle volume concentration v_p of 4%, the property ratios of which are the density ratio of 1.078 calculated from $\rho_e/\rho_m = [(1-v_p)\rho_m + v_p\rho_p]/\rho_m$, the specific heat ratio of 0.914 calculated from $C_{pe}/C_{pm} = \{[(1-v_p)(\rho C_p)_m + v_p(\rho C_p)_p]/[(1-v_p)\rho_m + v_p\rho_p]\}/C_{pm}$, and the thermal conductivity ratio of 1.120 calculated from $k_e/k_m = 1+3v_p$. It can be seen from Fig. 1 that, unlike the constant flow velocity comparison [Fig. 1(b)] and the constant pumping power comparison [Fig. 1(c)] where the flow velocity ratio V_e/V_m is either equal to or close to unity, the flow velocity ratio V_e/V_m for the constant Reynolds number comparison [Fig. 1(a)] is generally greater than unity. In other words, the flow velocity for the nanofluid is generally greater than that for the base fluid with the constant Reynolds number comparison. Therefore, the net result for the constant Reynolds number comparison is a combination of the nanofluid property effect and the flow velocity effect. Since the flow velocity for the nanofluid is higher than that for the base fluid, the constant Reynolds number comparison gives the nanofluid an advantage over the base fluid. If the nanofluid is to be pumped at a higher velocity in order to maintain the same Reynolds number as the base fluid, so could the base fluid be pumped at that velocity to obtain a better heat transfer coefficient. In that case, the base fluid might approach or exceed the performance of the nanofluid but this condition is not reflected in the constant Reynolds number comparison. The experimental results[8] of ten studies from the literature all showed $0.99 < r_{\mathrm{Re}} < 1.63$ while 70% of the cases had $r_V < 1$.

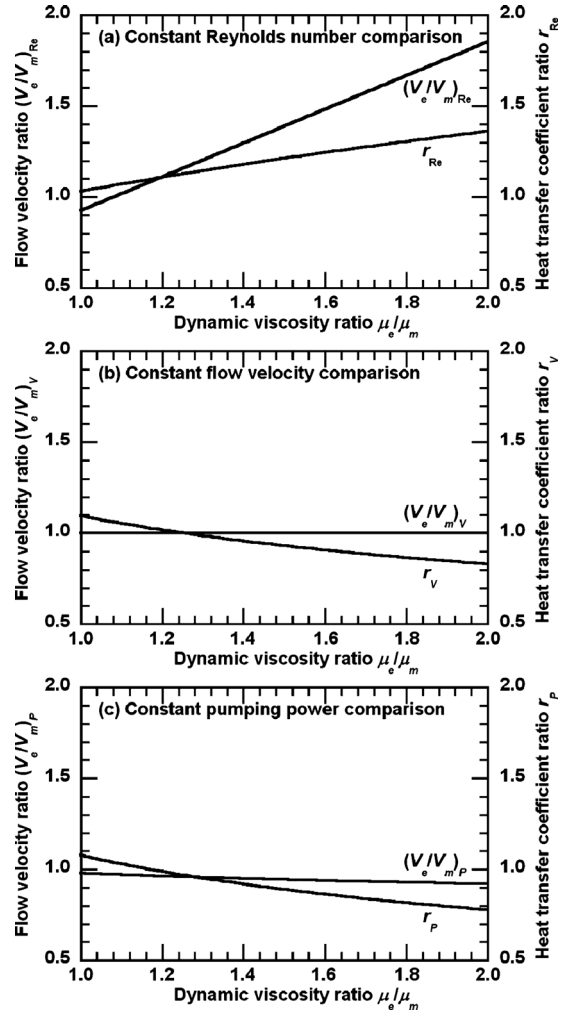

FIG. 1. Flow velocity ratio and heat transfer coefficient ratio as a function of dynamic viscosity ratio.

From an alternate viewpoint, the criteria for the three heat transfer enhancements of nanofluids over base fluids $r = h_e/h_m > 1$ can be translated into relationships between the dynamic viscosity ratio μ_e/μ_m and the thermal conductivity ratio k_e/k_m. These relationships, obtained by letting $r = h_e/h_m = 1$ for the three comparing bases, are shown in Fig. 2. The curves in Figs. 2(a)–2(c) represent the criteria for $r_{\mathrm{Re}} = 1$, $r_V = 1$, and $r_P = 1$, respectively, and they are applicable to any nanofluid and base fluid combination where the density ratio ρ_e/ρ_m and the specific heat ratio C_{pe}/C_{pm} are close to unity. It is seen that the nanofluid heat transfer enhancement increases in the region *above* the curve in Fig. 2(a) for the constant Reynolds number comparison and it increases *below* the curves in Figs. 2(b) and 2(c) for the constant flow velocity comparison and the constant pumping power comparison.

In the results shown in Fig. 2, the flow velocity differences discussed above have other disturbing implications as follows. As shown in Fig. 2(a) and in Eq. (4) for the constant Reynolds number comparison, at any value of thermal con-

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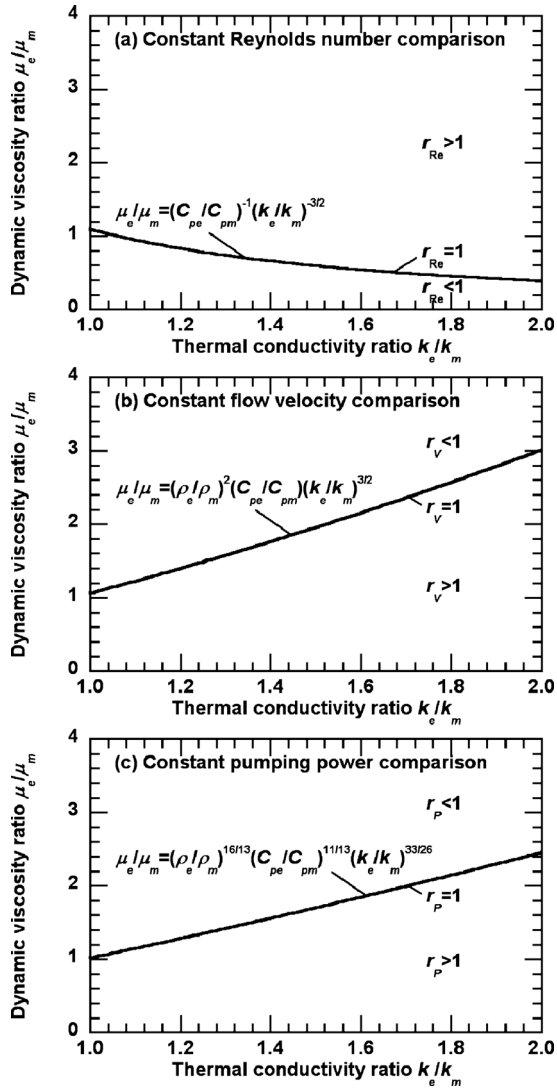

FIG. 2. Criteria for a beneficial nanofluid.

ductivity ratio k_e/k_m, the heat transfer coefficient ratio h_e/h_m increases as the dynamic viscosity ratio μ_e/μ_m increases. This result is opposite to those from the constant flow velocity comparison and the constant pumping power comparison of Figs. 2(b) and 2(c). It is in contradiction to common sense, and the reason is again rooted in the flow velocity effect. The constant Reynolds number comparison completely ignores the pumping power penalty associated with pumping the higher viscosity nanofluid at a higher flow velocity to reach the same Reynolds number as the base fluid. If the same pumping power or the same flow velocity for the nanofluid were used for the base fluid in Fig. 2(a), the heat transfer enhancement would be much less dramatic if it existed at all. In fact, as can be seen from Eq. (4) and the comparison of experimental studies,[8] it is almost certain that the heat transfer coefficient ratio h_e/h_m based on the constant Reynolds number comparison is greater than unity for any nanofluid with typical specific heat, dynamic viscosity, and thermal conductivity ratios, which, of course, is unrealistic. The energy available to pump a nanofluid must be considered to be available to pump the base fluid as well.

In contrast to the unrealistic figure of merit r_{Re} associated with the constant Reynolds number comparison, the figure of merit r_P for constant pumping power comparison represents the most unambiguous. With this comparison principle, the heat transfer coefficient of a nanofluid is compared to that of the base fluid under the condition that the nanofluid and the base fluid use the same amount of pumping power. As discussed above, this comparison means that the flow velocity for the nanofluid is slightly lower than that for the base fluid, as shown in Eq. (8), and gives the most conservative criterion for a beneficial nanofluid. Based on the physical mechanisms behind these comparisons, the constant pumping power comparison should be used for comparing the heat transfer coefficient of a nanofluid with that of the base fluid.

The constant flow velocity comparison is basically a Mouromtseff number comparison. The results for the constant flow velocity comparison of Fig. 2(b) have a similar trend to that for the constant pumping power comparison of Fig. 2(c) but the constant flow velocity comparison is less restrictive. The pumping power required for a nanofluid will be higher than that for the base fluid under the constant flow velocity comparison due to the increased density and dynamic viscosity of the nanofluid. However, as noted previously, the flow velocity ratio for the constant pumping power comparison is close to unity, and thus the curves of Figs. 2(b) and 2(c) are very close in magnitude. Therefore, from a practical viewpoint, when the flow system under consideration is designed for a certain maximum flow rate and when the flow pumping power is only a small portion of the total power consumed, the constant flow velocity comparison provides a quite accurate picture of the effect of the nanofluid properties on the heat transfer performance compared to the base fluid.

In conclusion, the criteria for comparing the fluid property-related heat transfer performance of a nanofluid to that of the base fluid under the bases of the constant Reynolds number, constant flow velocity, and constant friction-related pumping power have been developed for turbulent flow. (Heat transfer rates of nanofluids and their base fluids in fully-developed laminar flow are independent of viscosity, which, as discussed, is the major deleterious property for turbulent flow.) Among the three comparisons, the constant pumping power comparison is the most unambiguous; the constant flow velocity comparison can give quite good results under certain conditions but the constant Reynolds number comparison distorts the physical situation, and therefore, should not be used.

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[1] I. E. Mouromtseff, Proc. IRE **30**, 190 (1942).
[2] L.-T. Yeh and R. C. Chu, *Thermal Management of Microelectronic Equipment* (ASME, New York, 2002).
[3] W. C. Williams, J. Buongiorno, and L.-W. Hu, Trans. Am. Nucl. Soc. **96**, 481 (2007).
[4] F. W. Dittus and L. M. K. Boelter, Univ. Calif. Publ. Eng. **2**, 443 (1930).
[5] B. S. Petukhov and V. N. Popov, High Temp. **1**, 69 (1963).
[6] V. V. Gnielinski, Forsch. Ingenieurwes. **41**, 8 (1975).
[7] H. Blasius, Forsch.-Arb. Ing.-Wes. **131**, 1 (1913).
[8] E. V. Timofeeva, D. S. Smith, W. Yu, D. M. France, D. Singh, and J. L. Routbort, Nanotechnology **21**, 215703 (2010).

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