Convective Heat Transfer of Nanofluids in Turbulent Flow

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Essential Aspects of Nanofluid Heat Transfer Applications

- Selection of nanofluids based on thermophysical properties
  - Comparison criteria of the thermophysical property-related heat transfer performance of nanofluids and their base fluids
- Design of heat transfer systems using nanofluids
  - Predictions of the heat transfer coefficients of nanofluids based on homogeneous fluid models by using nanofluid effective thermophysical properties
- Effectiveness of nanofluids compared to their base fluids
  - Enhancements of the heat transfer coefficients of nanofluids over their base fluids

Effective Density of Nanofluids

- Mass
  - Base fluid mass m_m, particle mass m_p, nanofluid mass m_e

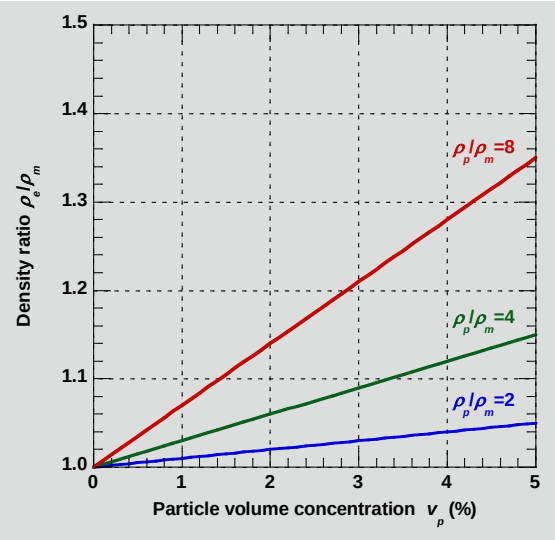
$$m_e = m_m + m_p$$

- Volume
  - Base fluid volume V_m, particle volume V_p, nanofluid volume V_e

$$V_e = V_m + V_p$$

- Density
  - Base fluid density ρ_m, particle density ρ_p, nanofluid density ρ_e

$$\rho_e = \frac{m_e}{V_e} = \frac{m_m + m_p}{V_e} = \frac{V_m \rho_m + V_p \rho_p}{V_e} = (1 - v_p)\rho_m + v_p \rho_p$$



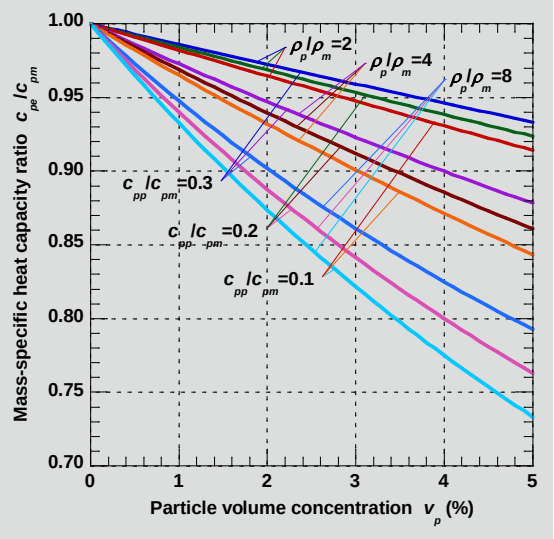
Effective Mass-Specific Heat Capacity of Nanofluids

- Heat input
  - Base fluid heat input q_m, particle heat input q_p, nanofluid heat input q_e

$$q_e = q_m + q_p$$

- Temperature change under thermal equilibrium
  - Base fluid temperature change ΔT_m, particle temperature change ΔT_p, nanofluid temperature change ΔT_e

$$\Delta T = \Delta T_m = \Delta T_p = \Delta T_e$$



- Mass-specific heat capacity
  - Base fluid mass-specific heat capacity c_{pm}, particle mass-specific heat capacity c_{pp}, nanofluid mass-specific heat capacity c_{pe}

$$c_{pe} = \frac{q_e}{m_e \Delta T} = \frac{V_m \rho_m c_{pm} + V_p \rho_p c_{pp}}{V_m \rho_m + V_p \rho_p} = \frac{(1-v_p)\rho_m c_{pm} + v_p \rho_p c_{pp}}{(1-v_p)\rho_m + v_p \rho_p}$$

Effective Dynamic Viscosity of Nanofluids

- Main influence factors
  - Particle volume concentration
  - Base fluid dynamic viscosity
  - Particle characteristic
    - Size and distribution, shape and orientation, aggregation and cluster, and Brownian motion
  - Particle-base fluid interfacial layer and nature
  - Nanofluid pH and additive
  - Nanofluid temperature
- Einstein equation — first-order approximation as a function of the base fluid dynamic viscosity μ_m and the particle volume concentration v_p

$$\mu_e = (1 + 2.5v_p)\mu_m$$

- Effective dynamic viscosity of nanofluids
  - Generally greater than the predictions of the Einstein equation
  - Most accurate and reliable approach – experimental measurements

Effective Thermal Conductivity of Nanofluids

- Main influence factors
  - Particle volume concentration
  - Base fluid thermal conductivity and particle thermal conductivity
  - Particle characteristic
    - Size and distribution, shape and orientation, aggregation and cluster, and Brownian motion
  - Particle-base fluid interfacial layer and nature
  - Nanofluid pH and additive
  - Nanofluid temperature
- Maxwell equation — first-order approximation as a function of the component thermal conductivities k_m and k_p and the particle volume concentration v_p

$$k_e = k_m + 3v_p \frac{k_p - k_m}{2k_m + k_p - v_p(k_p - k_m)} k_m$$

- Effective thermal conductivity of nanofluids
  - Imperfect match for the predictions of the Maxwell equation
  - Most accurate and reliable approach – experimental measurements

Primary Elements of Nanofluid Heat Transfer Comparisons

- Comparison bases
  - Unchanged variables that specify the conditions under which the heat transfer performance of nanofluids is compared to that of their base fluids
- Figures of merit
  - Quantities that are used to characterize the heat transfer performance and to determine the utility of nanofluids relative to their base fluid
- Basic assumptions (theoretical analyses)
  - Physical models of the interested system
  - Mathematical models of the heat transfer coefficient
  - Mathematical models of the Fanning friction factor
- Beneficial nanofluids
  - Figure-of-merit ratio of a nanofluid over its base fluid — larger than unity (>1)

Comparison Bases: Nondimensional Reynolds Number

- The Reynolds number is widely used for
  - Dimensional analysis of fluid dynamics
  - Determination of dynamic similitude between different experimental cases
  - Characterization of different flow regimes (laminar, transition, and turbulent)
  - Heat transfer coefficient prediction (almost in every heat transfer coefficient prediction equation based on theoretical analyses or experimental data or both)
- The Reynolds number is the most common choice of the comparison basis in the literature
- Unfortunately, the Reynolds number (the ratio of inertial forces to viscous forces for given flow conditions of a fluid) is an inappropriate basis for comparing the heat transfer performance of different fluids since the constant Reynolds number
  - Requires that the flow velocity for the more viscous nanofluid be higher than that for the less viscous base fluid (an advantage for the nanofluid over the base fluid)
  - Implies that the nanofluid convective heat transfer coefficient enhancement increases as its dynamic viscosity increases (in contradiction to common sense)
  - Ignores the pumping power penalty associated with pumping the more viscous nanofluid at a higher flow velocity

Comparison Bases: Pumping Power and Flow Velocity

- If a certain amount of energy is available to pump a nanofluid, it must be considered to be available to pump the base fluid as well
- Therefore, the same pumping power is a logical choice of the comparison basis, which
  - Compares different fluids under the condition that they use the same amount of pumping power
  - Connects two essential aspects: the heat transfer coefficient enhancement and the pumping power penalty
  - Requires that the flow velocity for the more viscous nanofluid be slightly lower than that for the less viscous base fluid
  - Is the most conservative and unambiguous comparison basis for a beneficial nanofluid
- As an alternative, the same flow velocity
  - Compares different fluids under the condition that their flow velocities are same
  - Requires that the pumping power for the more viscous nanofluid be slightly higher than that for the less viscous base fluid
  - Provides a quite accurate picture when the flow system under consideration is designed for a certain maximum flow rate and when the flow pumping power is only a small portion of the total power consumed

Simple Figures of Merit

- The nondimensional Nusselt number — a common but inappropriate choice of the simple figure of merit because, when comparing two or more fluids with different thermal conductivities
  - It is the convective heat transfer coefficient but not the Nusselt number (the convective-to-conductive heat transfer coefficient ratio) that matters
  - A fluid with a lower Nusselt number might have a higher convective heat transfer coefficient if it has a higher thermal conductivity (such as in the case of a nanofluid)
- The heat transfer coefficient — a direct expression of the convective heat transfer capability of a fluid and therefore an appropriate simple figure of merit for comparing
  - Theoretical predictions
  - Experimental data
- As an alternative, the Mouromtseff number
  - Is primarily used for the comparison of theoretical predictions of different fluids
  - Is equivalent to the heat transfer coefficient when expressed in terms of the ratios between the compared fluids for the constant flow velocity
  - Can usually be defined in the following general form $$\mathrm{Mo} = \rho^{\alpha_1} c_p^{\alpha_2} \mu^{\alpha_3} k^{\alpha_4}$$

Figures of Merit: Other Influence Factors

- Pumping power effects
  - Heat transfer coefficient enhancement — a positive effect
  - Pumping power increase — a negative effect
  - Combined figure of merit

$$\mathrm{FOM} = (h_e / h_m) / (P_e / P_m)$$

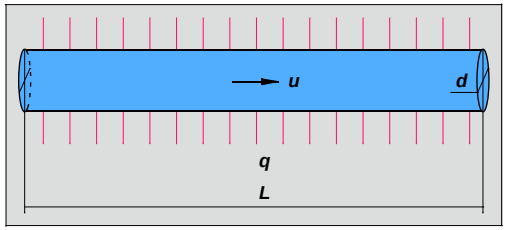
- Mass-specific heat capacity effects
  - Mass-specific heat capacity — the capacity for a fluid to continuously remove the heat from a heat source by keeping their temperature difference high
  - Combined figure of merit

$$\mathrm{FOM} = c_p h / P$$

- Temperature effects (for systems confined to certain wall or fluid temperatures)
  - Two logical comparison bases
    - Fluid temperature change (involving the effect of the fluid mass-specific heat capacity)
    - Wall-fluid temperature difference (equivalent to the heat transfer coefficient)
  - For the design purpose, the pumping power is usually employed as the figure of merit

Basic Assumptions

- Physical model — one of the most commonly-encountered engineering heat transfer problems
  - Forced single-phase internal flow through a circular tube with the flow channel diameter of *d* and the flow channel length of *L*



- Heat transfer coefficient for turbulent flow
  - Dittus-Boelter equation (for simplicity)

$$h = \begin{cases} 0.023\,\mathrm{Re}^{4/5}\,\mathrm{Pr}^{1/3}(k/d) & \text{for cooling} \\ 0.023\,\mathrm{Re}^{4/5}\,\mathrm{Pr}^{2/5}(k/d) & \text{for heating} \end{cases}$$

- Fanning friction factor for turbulent flow

$$f = \begin{cases} 0.046\,\mathrm{Re}^{-1/5} & \text{classical equation} \\ 0.0791\,\mathrm{Re}^{-1/4} & \text{Blasius equation} \end{cases}$$

Figure-of-Merit Ratios: Cooling with the Classical Equation for the Fanning Friction Factor

- Same flow velocity comparison — $r_1>r_2>r_3$ due to $Pe/Pm>1$ and $c_{pe}/c_{pm}<1$
- Same pumping power comparison — $r_4=r_5>r_6$ due to $Pe/Pm=1$ and $c_{pe}/c_{pm}<1$
- Same fluid temperature change — generally $r_7<1$ and unrelated to k (unsuitable)

Flow	Comparison Basis	Figure-of-Merit Ratio
Fully-developed internal turbulent flow for cooling with the classical equation for the Fanning friction factor $h = 0.023\,\mathrm{Re}^{4/5}\,\mathrm{Pr}^{1/3}(k/d)$ $= \frac{0.023u^{4/5}}{d^{1/5}}\frac{\rho^{4/5}c_p^{1/3}k^{2/3}}{\mu^{7/15}}$ $= \frac{0.0487P^{2/7}}{d^{3/7}L^{2/7}}\frac{\rho^{4/7}c_p^{1/3}k^{2/3}}{\mu^{11/21}}$ $f = 0.046\,\mathrm{Re}^{-1/5}$ $P = (\pi/2)dLu^3\rho f$ $= 0.0723d^{4/5}Lu^{14/5}\rho^{4/5}\mu^{1/5}$ $= \frac{0.1421L\dot{q}^{14/5}}{d^{24/5}(T_{fin}-T_{fout})^{14/5}}\frac{\mu^{1/5}}{\rho^2c_p^{14/5}}$ $= 39159d^{3/2}Lh^{7/2}\frac{\mu^{11/6}}{\rho^2c_p^{7/6}k^{7/3}}$	Flow velocity u	$r_1 = \frac{h_e}{h_m} = \frac{\mathrm{Mo}_e}{\mathrm{Mo}_m} = \left(\frac{\rho_e}{\rho_m}\right)^{4/5}\left(\frac{c_{pe}}{c_{pm}}\right)^{1/3}\left(\frac{\mu_e}{\mu_m}\right)^{-7/15}\left(\frac{k_e}{k_m}\right)^{2/3}$
	Flow velocity u	$r_2 = \frac{h_e/h_m}{P_e/P_m} = \left(\frac{c_{pe}}{c_{pm}}\right)^{1/3}\left(\frac{\mu_e}{\mu_m}\right)^{-2/3}\left(\frac{k_e}{k_m}\right)^{2/3}$
	Flow velocity u	$r_3 = \frac{c_{pe}h_e/P_e}{c_{pm}h_m/P_m} = \left(\frac{c_{pe}}{c_{pm}}\right)^{4/3}\left(\frac{\mu_e}{\mu_m}\right)^{-2/3}\left(\frac{k_e}{k_m}\right)^{2/3}$
	Pump power P	$r_4 = \frac{h_e}{h_m} = \left(\frac{\rho_e}{\rho_m}\right)^{4/7}\left(\frac{c_{pe}}{c_{pm}}\right)^{1/3}\left(\frac{\mu_e}{\mu_m}\right)^{-11/21}\left(\frac{k_e}{k_m}\right)^{2/3}$
	Pump power P	$r_5 = \frac{h_e/h_m}{P_e/P_m} = \frac{h_e}{h_m} = \left(\frac{\rho_e}{\rho_m}\right)^{4/7}\left(\frac{c_{pe}}{c_{pm}}\right)^{1/3}\left(\frac{\mu_e}{\mu_m}\right)^{-11/21}\left(\frac{k_e}{k_m}\right)^{2/3}$
	Pump power P	$r_6 = \frac{c_{pe}h_e/P_e}{c_{pm}h_m/P_m} = \frac{c_{pe}h_e}{c_{pm}h_m} = \left(\frac{\rho_e}{\rho_m}\right)^{4/7}\left(\frac{c_{pe}}{c_{pm}}\right)^{4/3}\left(\frac{\mu_e}{\mu_m}\right)^{-11/21}\left(\frac{k_e}{k_m}\right)^{2/3}$
	Temperature change $T_{fin}-T_{fout}$	$r_7 = \frac{P_m}{P_e} = \left(\frac{\rho_e}{\rho_m}\right)^{2}\left(\frac{c_{pe}}{c_{pm}}\right)^{14/5}\left(\frac{\mu_e}{\mu_m}\right)^{-1/5}$
	Temperature difference T_f-T_w	$r_8 = \frac{P_m}{P_e} = \left(\frac{\rho_e}{\rho_m}\right)^{2}\left(\frac{c_{pe}}{c_{pm}}\right)^{7/6}\left(\frac{\mu_e}{\mu_m}\right)^{-11/6}\left(\frac{k_e}{k_m}\right)^{7/3}$

Figure-of-Merit Ratios: Cooling with the Blasius Equation for the Fanning Friction Factor

- Same flow velocity comparison — $r_9 > r_{10} > r_{11}$ due to $Pe/Pm > 1$ and $c_{pe}/c_{pm} < 1$
- Same pumping power comparison — $r_{12} = r_{13} > r_{14}$ due to $Pe/Pm = 1$ and $c_{pe}/c_{pm} < 1$
- Same fluid temperature change — generally $r_{15} < 1$ and unrelated to k (unsuitable)
- Same flow velocity comparison — always $r_1 = r_9$, unrelated to f

Flow	Comparison Basis	Figure-of-Merit Ratio
Fully-developed internal turbulent flow for cooling with the Blasius equation for the Fanning friction factor $h = 0.023\,\mathrm{Re}^{4/5}\,\mathrm{Pr}^{1/3}(k/d) = \frac{0.023u^{4/5}}{d^{1/5}}\frac{\rho^{4/5}c_p^{1/3}k^{2/3}}{\mu^{7/15}} = \frac{0.0422P^{16/55}}{d^{23/55}L^{16/55}}\frac{\rho^{32/55}c_p^{1/3}k^{2/3}}{\mu^{89/165}}$ $f = 0.0791\,\mathrm{Re}^{-1/4}$ $P = (\pi/2)dLu^3\rho f = 0.1242d^{3/4}Lu^{11/4}\rho^{3/4}\mu^{1/4} = \frac{0.2414L\dot{q}^{11/4}}{d^{19/4}(T_{fin}-T_{fout})^{11/4}}\frac{\mu^{1/4}}{\rho^2c_p^{11/4}} = 53193d^{23/16}Lh^{55/16}\frac{\mu^{89/48}}{\rho^2c_p^{55/48}k^{55/24}}$	Flow velocity u	$r_9 = \frac{h_e}{h_m} = \frac{\mathrm{Mo}_e}{\mathrm{Mo}_m} = \left(\frac{\rho_e}{\rho_m}\right)^{4/5}\left(\frac{c_{pe}}{c_{pm}}\right)^{1/3}\left(\frac{\mu_e}{\mu_m}\right)^{-7/15}\left(\frac{k_e}{k_m}\right)^{2/3}$
	Flow velocity u	$r_{10} = \frac{h_e/h_m}{P_e/P_m} = \left(\frac{\rho_e}{\rho_m}\right)^{1/20}\left(\frac{c_{pe}}{c_{pm}}\right)^{1/3}\left(\frac{\mu_e}{\mu_m}\right)^{-43/60}\left(\frac{k_e}{k_m}\right)^{2/3}$
	Flow velocity u	$r_{11} = \frac{c_{pe}h_e/P_e}{c_{pm}h_m/P_m} = \left(\frac{\rho_e}{\rho_m}\right)^{1/20}\left(\frac{c_{pe}}{c_{pm}}\right)^{4/3}\left(\frac{\mu_e}{\mu_m}\right)^{-43/60}\left(\frac{k_e}{k_m}\right)^{2/3}$
	Pump power P	$r_{12} = \frac{h_e}{h_m} = \left(\frac{\rho_e}{\rho_m}\right)^{32/55}\left(\frac{c_{pe}}{c_{pm}}\right)^{1/3}\left(\frac{\mu_e}{\mu_m}\right)^{-89/165}\left(\frac{k_e}{k_m}\right)^{2/3}$
	Pump power P	$r_{13} = \frac{h_e/h_m}{P_e/P_m} = \frac{h_e}{h_m} = \left(\frac{\rho_e}{\rho_m}\right)^{32/55}\left(\frac{c_{pe}}{c_{pm}}\right)^{1/3}\left(\frac{\mu_e}{\mu_m}\right)^{-89/165}\left(\frac{k_e}{k_m}\right)^{2/3}$
	Pump power P	$r_{14} = \frac{c_{pe}h_e/P_e}{c_{pm}h_m/P_m} = \frac{c_{pe}h_e}{c_{pm}h_m} = \left(\frac{\rho_e}{\rho_m}\right)^{32/55}\left(\frac{c_{pe}}{c_{pm}}\right)^{4/3}\left(\frac{\mu_e}{\mu_m}\right)^{-89/165}\left(\frac{k_e}{k_m}\right)^{2/3}$
	Temperature change $T_{fin} - T_{fout}$	$r_{15} = \frac{P_m}{P_e} = \left(\frac{\rho_e}{\rho_m}\right)^{2}\left(\frac{c_{pe}}{c_{pm}}\right)^{11/4}\left(\frac{\mu_e}{\mu_m}\right)^{-1/4}$
	Temperature difference $T_f - T_w$	$r_{16} = \frac{P_m}{P_e} = \left(\frac{\rho_e}{\rho_m}\right)^{2}\left(\frac{c_{pe}}{c_{pm}}\right)^{55/48}\left(\frac{\mu_e}{\mu_m}\right)^{-89/48}\left(\frac{k_e}{k_m}\right)^{55/24}$

Figure-of-Merit Ratios: Heating with the Classical Equation for the Fanning Friction Factor

- Same flow velocity comparison — $r_{17}>r_{18}>r_{19}$ due to $Pe/Pm>1$ and $c_{pe}/c_{pm}<1$
- Same pumping power comparison — $r_{20}=r_{21}>r_{22}$ due to $Pe/Pm=1$ and $c_{pe}/c_{pm}<1$
- Same fluid temperature change — generally $r_{23}<1$ and unrelated to k (unsuitable)

Flow	Comparison Basis	Figure-of-Merit Ratio
Fully-developed internal turbulent flow for heating with the classical equation for the Fanning friction factor $h = 0.023\,\mathrm{Re}^{4/5}\,\mathrm{Pr}^{2/5}(k/d)$ $= \frac{0.023u^{4/5}}{d^{1/5}}\frac{\rho^{4/5}c_p^{2/5}k^{3/5}}{\mu^{2/5}}$ $= \frac{0.0487P^{2/7}}{d^{3/7}L^{2/7}}\frac{\rho^{4/7}c_p^{2/5}k^{3/5}}{\mu^{16/35}}$ $f = 0.046\,\mathrm{Re}^{-1/5}$ $P = (\pi/2)dLu^3\rho f$ $= 0.0723d^{4/5}Lu^{14/5}\rho^{4/5}\mu^{1/5}$ $= \frac{0.1421L\dot{q}^{14/5}}{d^{24/5}(T_{fout}-T_{fin})^{14/5}}\frac{\mu^{1/5}}{\rho^2c_p^{14/5}}$ $= 39159d^{3/2}Lh^{7/2}\frac{\mu^{8/5}}{\rho^2c_p^{7/5}k^{21/10}}$	Flow velocity u	$r_{17} = \frac{h_e}{h_m} = \frac{\mathrm{Mo}_e}{\mathrm{Mo}_m} = \left(\frac{\rho_e}{\rho_m}\right)^{4/5}\left(\frac{c_{pe}}{c_{pm}}\right)^{2/5}\left(\frac{\mu_e}{\mu_m}\right)^{-2/5}\left(\frac{k_e}{k_m}\right)^{3/5}$
	Flow velocity u	$r_{18} = \frac{h_e/h_m}{P_e/P_m} = \left(\frac{c_{pe}}{c_{pm}}\right)^{2/5}\left(\frac{\mu_e}{\mu_m}\right)^{-3/5}\left(\frac{k_e}{k_m}\right)^{3/5}$
	Flow velocity u	$r_{19} = \frac{h_e c_{pe}/P_e}{h_m c_{pm}/P_m} = \left(\frac{c_{pe}}{c_{pm}}\right)^{7/5}\left(\frac{\mu_e}{\mu_m}\right)^{-3/5}\left(\frac{k_e}{k_m}\right)^{3/5}$
	Pumping power P	$r_{20} = \frac{h_e}{h_m} = \left(\frac{\rho_e}{\rho_m}\right)^{4/7}\left(\frac{c_{pe}}{c_{pm}}\right)^{2/5}\left(\frac{\mu_e}{\mu_m}\right)^{-16/35}\left(\frac{k_e}{k_m}\right)^{3/5}$
	Pumping power P	$r_{21} = \frac{h_e/h_m}{P_e/P_m} = \frac{h_e}{h_m} = \left(\frac{\rho_e}{\rho_m}\right)^{4/7}\left(\frac{c_{pe}}{c_{pm}}\right)^{2/5}\left(\frac{\mu_e}{\mu_m}\right)^{-16/35}\left(\frac{k_e}{k_m}\right)^{3/5}$
	Pumping power P	$r_{22} = \frac{h_e c_{pe}/P_e}{h_m c_{pm}/P_m} = \frac{c_{pe}h_e}{c_{pm}h_m} = \left(\frac{\rho_e}{\rho_m}\right)^{4/7}\left(\frac{c_{pe}}{c_{pm}}\right)^{7/5}\left(\frac{\mu_e}{\mu_m}\right)^{-16/35}\left(\frac{k_e}{k_m}\right)^{3/5}$
	Temperature change $T_{fout}-T_{fin}$	$r_{23} = \frac{P_m}{P_e} = \left(\frac{\rho_e}{\rho_m}\right)^{2}\left(\frac{c_{pe}}{c_{pm}}\right)^{14/5}\left(\frac{\mu_e}{\mu_m}\right)^{-1/5}$
	Temperature difference T_w-T_f	$r_{24} = \frac{P_m}{P_e} = \left(\frac{\rho_e}{\rho_m}\right)^{2}\left(\frac{c_{pe}}{c_{pm}}\right)^{7/5}\left(\frac{\mu_e}{\mu_m}\right)^{-8/5}\left(\frac{k_e}{k_m}\right)^{21/10}$

Figure-of-Merit Ratios: Heating with the Blasius Equation for the Fanning Friction Factor

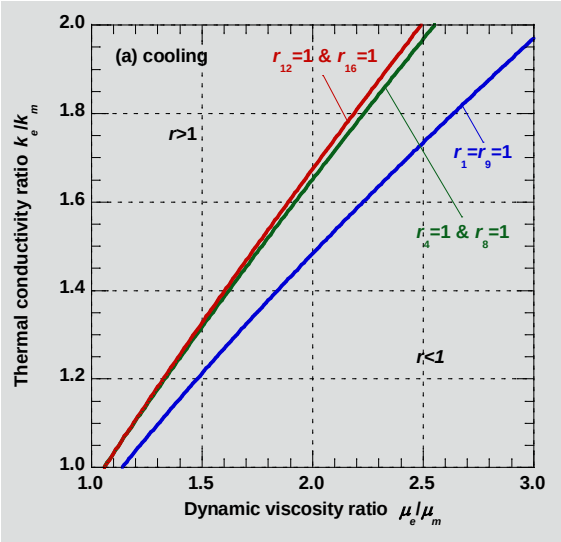
- Same flow velocity comparison — $r_{25}>r_{26}>r_{27}$ due to $Pe/Pm>1$ and $c_{pe}/c_{pm}<1$
- Same pumping power comparison — $r_{28}=r_{29}>r_{30}$ due to $Pe/Pm=1$ and $c_{pe}/c_{pm}<1$
- Same fluid temperature change — generally $r_{31}<1$ and unrelated to k (unsuitable)
- Same flow velocity comparison — always $r_{17}=r_{25}$, unrelated to f

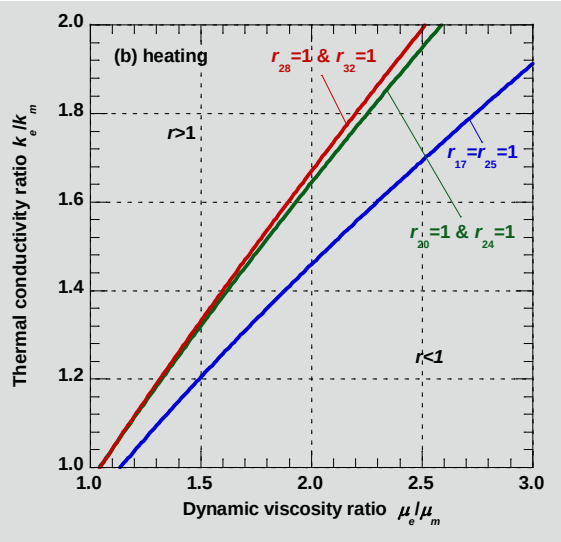
Flow	Comparison Basis	Figure-of-Merit Ratio
Fully-developed internal turbulent flow for heating with the Blasius equation for the Fanning friction factor $h = 0.023\,\mathrm{Re}^{4/5}\,\mathrm{Pr}^{2/5}(k/d)$ $= \frac{0.023u^{4/5}}{d^{1/5}}\frac{\rho^{4/5}c_p^{2/5}k^{3/5}}{\mu^{2/5}}$ $= \frac{0.0422P^{16/55}}{d^{23/55}L^{16/55}}\frac{\rho^{32/55}c_p^{2/5}k^{3/5}}{\mu^{26/55}}$ $f = 0.0791\,\mathrm{Re}^{-1/4}$ $P = (\pi/2)dLu^3\rho f$ $= 0.1242d^{3/4}Lu^{11/4}\rho^{3/4}\mu^{1/4}$ $= \frac{0.2414L\dot{q}^{11/4}}{d^{19/4}(T_{fout}-T_{fin})^{11/4}}\frac{\mu^{1/4}}{\rho^2c_p^{11/4}}$ $= 53193d^{23/16}Lh^{55/16}\frac{\mu^{13/8}}{\rho^2c_p^{11/8}k^{33/16}}$	Flow velocity u	$r_{25} = \frac{h_e}{h_m} = \frac{\mathrm{Mo}_e}{\mathrm{Mo}_m} = \left(\frac{\rho_e}{\rho_m}\right)^{4/5}\left(\frac{c_{pe}}{c_{pm}}\right)^{2/5}\left(\frac{\mu_e}{\mu_m}\right)^{-2/5}\left(\frac{k_e}{k_m}\right)^{3/5}$
	Flow velocity u	$r_{26} = \frac{h_e/h_m}{P_e/P_m} = \left(\frac{\rho_e}{\rho_m}\right)^{1/20}\left(\frac{c_{pe}}{c_{pm}}\right)^{2/5}\left(\frac{\mu_e}{\mu_m}\right)^{-13/20}\left(\frac{k_e}{k_m}\right)^{3/5}$
	Flow velocity u	$r_{27} = \frac{h_ec_{pe}/P_e}{h_mc_{pm}/P_m} = \left(\frac{\rho_e}{\rho_m}\right)^{1/20}\left(\frac{c_{pe}}{c_{pm}}\right)^{7/5}\left(\frac{\mu_e}{\mu_m}\right)^{-13/20}\left(\frac{k_e}{k_m}\right)^{3/5}$
	Pumping power P	$r_{28} = \frac{h_e}{h_m} = \left(\frac{\rho_e}{\rho_m}\right)^{32/55}\left(\frac{c_{pe}}{c_{pm}}\right)^{2/5}\left(\frac{\mu_e}{\mu_m}\right)^{-26/55}\left(\frac{k_e}{k_m}\right)^{3/5}$
	Pumping power P	$r_{29} = \frac{h_e/h_m}{P_e/P_m} = \frac{h_e}{h_m} = \left(\frac{\rho_e}{\rho_m}\right)^{32/55}\left(\frac{c_{pe}}{c_{pm}}\right)^{2/5}\left(\frac{\mu_e}{\mu_m}\right)^{-26/55}\left(\frac{k_e}{k_m}\right)^{3/5}$
	Pumping power P	$r_{30} = \frac{h_ec_{pe}/P_e}{h_mc_{pm}/P_m} = \frac{c_{pe}h_e}{c_{pm}h_m} = \left(\frac{\rho_e}{\rho_m}\right)^{32/55}\left(\frac{c_{pe}}{c_{pm}}\right)^{7/5}\left(\frac{\mu_e}{\mu_m}\right)^{-26/55}\left(\frac{k_e}{k_m}\right)^{3/5}$
	Temperature change $T_{fout}-T_{fin}$	$r_{31} = \frac{P_m}{P_e} = \left(\frac{\rho_e}{\rho_m}\right)^{2}\left(\frac{c_{pe}}{c_{pm}}\right)^{11/4}\left(\frac{\mu_e}{\mu_m}\right)^{-1/4}$
	Temperature difference T_w-T_f	$r_{32} = \frac{P_m}{P_e} = \left(\frac{\rho_e}{\rho_m}\right)^{2}\left(\frac{c_{pe}}{c_{pm}}\right)^{11/8}\left(\frac{\mu_e}{\mu_m}\right)^{-13/8}\left(\frac{k_e}{k_m}\right)^{33/16}$

Figure-of-Merit Ratios: Other Features

- For the same comparison basis, the differences of the figure-of-merit ratios are insignificant between cooling and heating and between the classical equation and the Blasius equation (verified by comparing the exponents of the property ratios)
- The figure-of-merit ratios can be translated into μ_e/μ_m and k_e/k_m plots (an example: an alumina-in-water nanofluid at 25 °C)
- *h* ratio under the constant *P* and *P* ratio under the constant *h* are the same curve

$$r_4^7 = r_8^2, \quad r_{12}^{55} = r_{16}^{16}, \quad r_{20}^7 = r_{24}^2, \quad r_{28}^{55} = r_{32}^{16}$$





Summary

- The comparison criteria can basically be classified into two main categories
  - To achieve the highest heat transfer coefficient under the constant pumping power
  - To reach the lowest pumping power under the constant heat transfer coefficient

- For practical applications, it is unnecessary to judge a beneficial nanofluid separately between cooling and heating and between the equations for the Fanning friction factor because of the insignificant differences resulted from them

- On the μ_e/μ_m and k_e/k_m plan, the unity figure-of-merit ratios for the heat transfer coefficient under the same pumping power comparison and for the pumping power under the same heat transfer coefficient comparison are the same curve — they are interchangeable for judging a beneficial nanofluid

Nanofluid Heat Transfer Coefficient Predictions

- Nanofluid heat transfer coefficients
  - A nanofluid is a complex two-phase and sometime multi-phase mixture
  - The closed solutions are difficult and sometime impossible to find for such complicated mathematical problems
  - The effective approach — treats a nanofluid as a homogeneous fluid of effective thermophysical properties
    - The basis for the comparison criteria (discussed previously)
    - To be examined through experimental data of the turbulent-flow convective heat transfer coefficients of nanofluids in the engineering literature
- Experimental database
  - Limited to convective heat transfer coefficients of nanofluids under
  - The condition of internal turbulent through a circular tube where
  - Nanofluid effective thermophysical properties, nanofluid temperatures, nanofluid flow velocities, and other necessary parameter are obtainable
  - Base fluids: water (H_2O) and 50/50 ethylene glycol/water ($C_2H_6O_2$/ H_2O)
  - Particles: alumina (Al_2O_3), copper (Cu), cupric oxide (CuO), diamond, multiwalled carbon nanotube (MWCNT), silicon carbide (SiC), silica (SiO_2), titania (TiO_2), and zirconia (ZrO_2)

Experimental Database

Author and Year	Nanofluid	Volume Concentration (%)	Channel (mm)	Note
Pak and Cho 1998	13-nm Al_2O_3 in H_2O	(1) 1.34 and (2) 2.78	10.66	Measured μ_e and k_e
Williams et al. 2008	46-nm Al_2O_3 in H_2O	(3) 0.9, (4) 1.8, and (5) 3.6	9.398	Fitted μ_e and k_e
Torii 2010	33-nm Al_2O_3 in H_2O	(6) 0.1, (7) 1.0, and (8) 5.0	3.96	Measured μ and k ratios
Xuan and Li 2003	<100-nm Cu in H_2O	(9) 0.3, (10) 0.5, (11) 0.8, (12) 1.0, (13) 1.2, (14) 1.5, and (15) 2.0	10	Measured μ_e and k_e
Fotukian and Esfahany 2010	30 – 50-nm CuO in H_2O	(16) 0.015, (17) 0.031, (18) 0.039, (19) 0.078, (20) 0.118, (21) 0.157, and (22) 0.236	5	Einstein equation for μ_e Maxwell equation for k_e
Torii 2010	47-nm CuO in H_2O	(23) 0.1, (24) 1.0, and (25) 5.0	3.96	Measured μ and k ratios
Torii and Yang 2009	2 – 10-nm diamond in H_2O	(26) 0.1, (27) 0.4, and (28) 1.0	3.96	Measured μ and k ratios
Torii 2010	2 – 10-nm diamond in H_2O	(29) 0.1, (30) 1.0, and (31) 5.0	3.96	Measured μ and k ratios
Liao and Liu 2009	10 – 20-nm×1 – 2-μm MWCNT in H_2O	(32) 0.243, (33) 0.486, and (34) 0.978	1.02	Measured μ and k ratios
Yu et al. 2009	170-nm SiC in H_2O	(35) 3.7	2.2733	Fitted μ_e and k_e
Timofeeva et al. 2010	16-nm SiC in H_2O	(36) 4.1	2.2733	Measured μ_e and k_e
Timofeeva et al. 2010	29-nm SiC in H_2O	(37) 4.1	2.2733	Measured μ_e and k_e
Timofeeva et al. 2010	66-nm SiC in H_2O	(38) 4.1	2.2733	Measured μ_e and k_e
Timofeeva et al. 2010	90-nm SiC in H_2O	(39) 4.1	2.2733	Measured μ_e and k_e
Ferrouillat et al. 2011	22-nm SiO_2 in H_2O	(40) 2.33, (41) 7.95, and (42) 18.93	4	Measured μ_e and k_e
Pak and Cho 1998	27-nm TiO_2 in H_2O	(43) 0.99, (44) 2.04, and (45) 3.16	10.66	Measured μ_e and k_e
Duangthongsuk and Wongwises 2009	21-nm TiO_2 in H_2O	(46) 0.2	8.13	Measured μ_e and k_e
Duangthongsuk and Wongwises 2010	21-nm TiO_2 in H_2O	(47) 0.2, (48) 0.6, (49) 1.0, (50) 1.5, and (51) 2.0	8.13	Measured μ_e and k_e
Williams et al. 2008	60-nm ZrO_2 in H_2O	(52) 0.2, (53) 0.5, and (54) 0.9	9.398	Fitted μ_e and k_e
Timofeeva et al. 2011	16-nm SiC in 50/50 $C_2H_6O_2/H_2O$	(55) 4.0	2.2733	Measured μ_e and k_e
Timofeeva et al. 2011	29-nm SiC in 50/50 $C_2H_6O_2/H_2O$	(56) 4.0	2.2733	Measured μ_e and k_e
Timofeeva et al. 2011	66-nm SiC in 50/50 $C_2H_6O_2/H_2O$	(57) 4.0	2.2733	Measured μ_e and k_e
Timofeeva et al. 2011	90-nm SiC in 50/50 $C_2H_6O_2/H_2O$	(58) 1.0 and (59) 4.0	2.2733	Measured μ_e and k_e

Nanofluid Heat Transfer Coefficient Comparisons

- The effective density and the effective mass-specific heat capacity are calculated from the nanofluid component thermophysical properties
- Three commonly-used equations for pure fluids are used for comparisons
  - Dittus-Boelter equation
  - Petukhov-Popov equation (f=[1.82logRe-1.64] $^{-2}$)

$$h = \frac{(f/8)\,\mathrm{Re}\,\mathrm{Pr}(k/d)}{1+3.4f+(11.7+1.8\,\mathrm{Pr}^{-1/3})(f/8)^{1/2}(\mathrm{Pr}^{2/3}-1)}$$

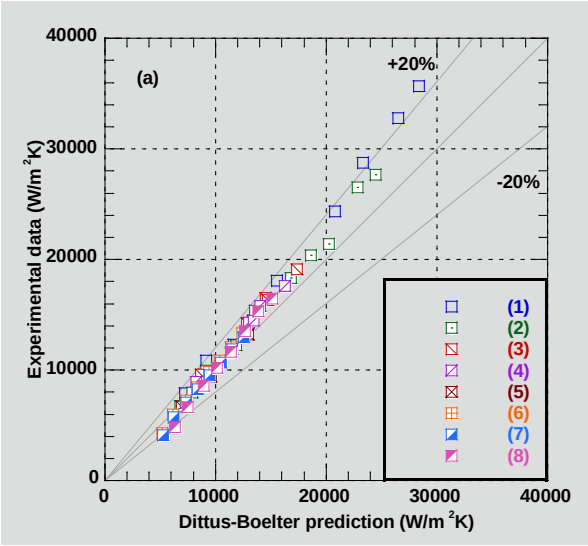
  - Gnielinski equation

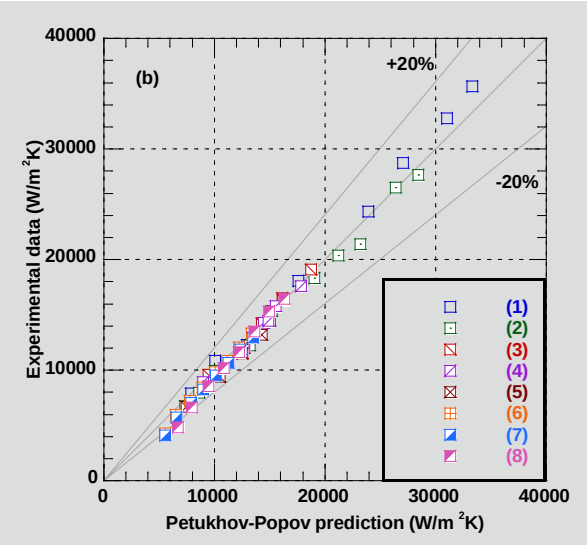
$$h = \frac{(f/8)(\mathrm{Re}-1000)\,\mathrm{Pr}(k/d)}{1+12.7(f/8)^{1/2}(\mathrm{Pr}^{2/3}-1)}$$

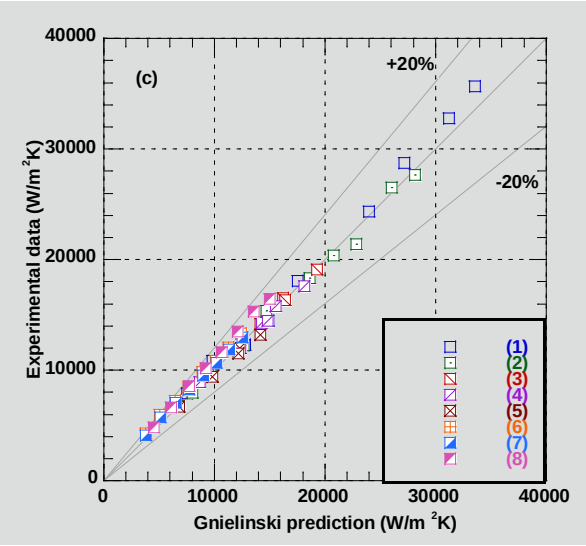
- The results can generally be divided into the following four groups

Nanofluid Heat Transfer Coefficient Comparisons: Group A

- All three equations predict most of the experimental data within the ±20% range
  - Alumina-in-water nanofluids (Al_2O_3 in H_2O)

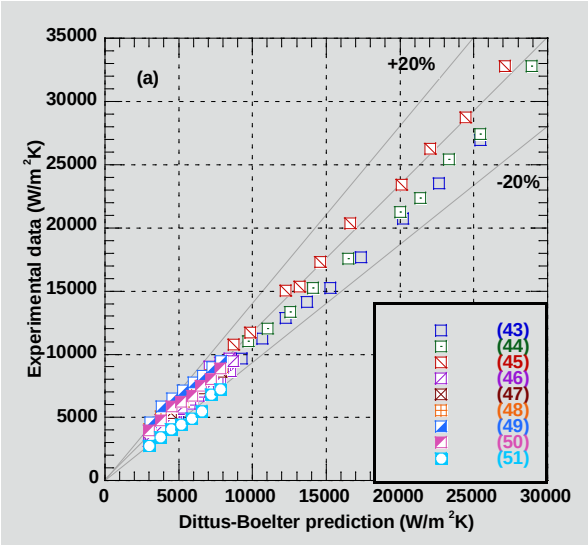


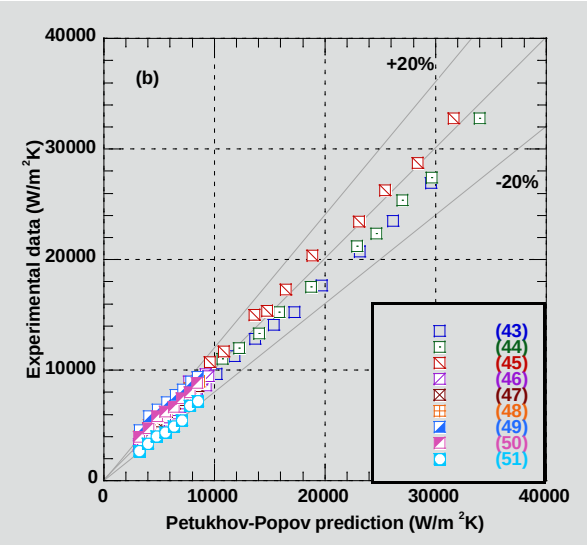


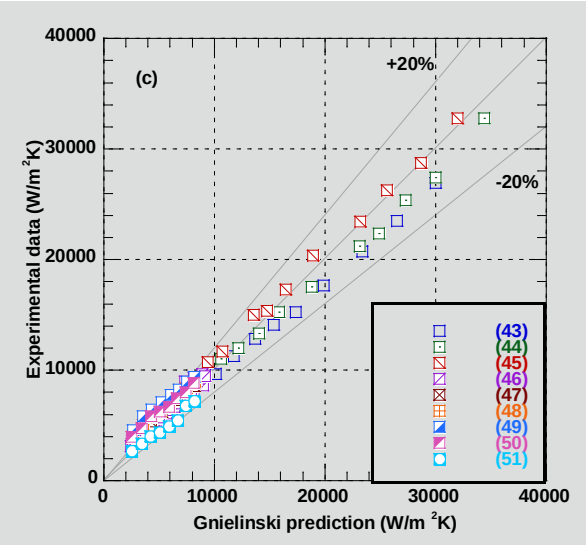


Nanofluid Heat Transfer Coefficient Comparisons: Group A

- All three equations predict most of the experimental data within the ±20% range
  - Titania-in-water nanofluids (TiO_2 in H_2O)

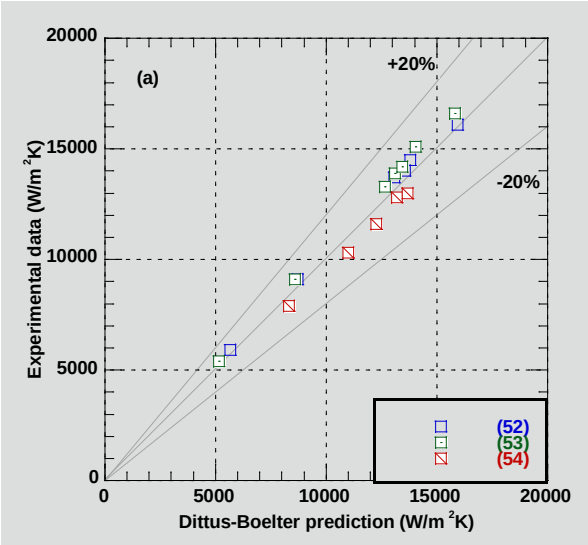


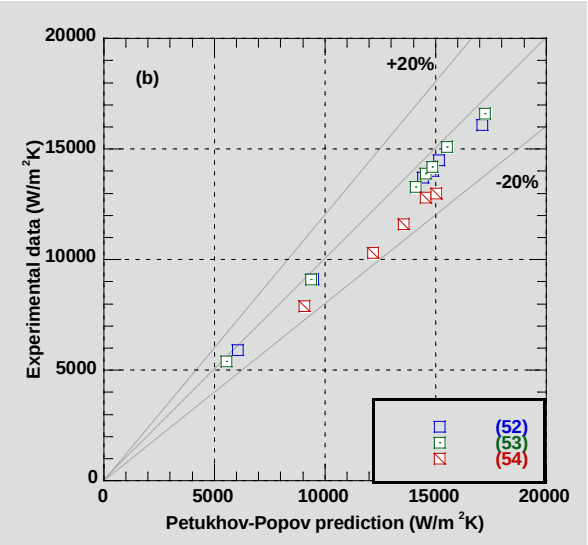


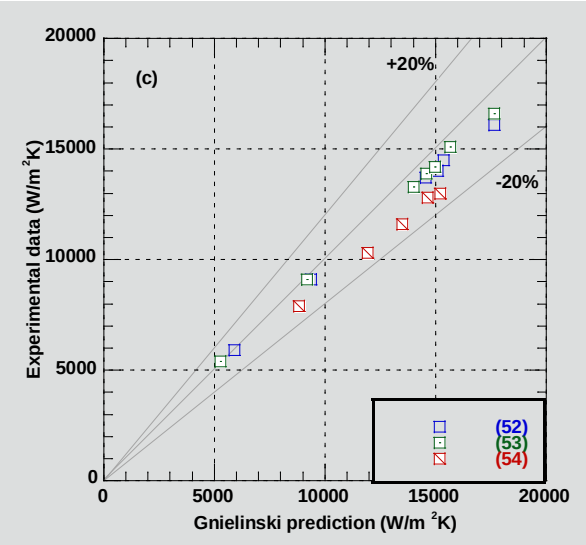


Nanofluid Heat Transfer Coefficient Comparisons: Group A

- All three equations predict most of the experimental data within the ±20% range
  - Zirconia-in-water nanofluids (ZrO_2 in H_2O)

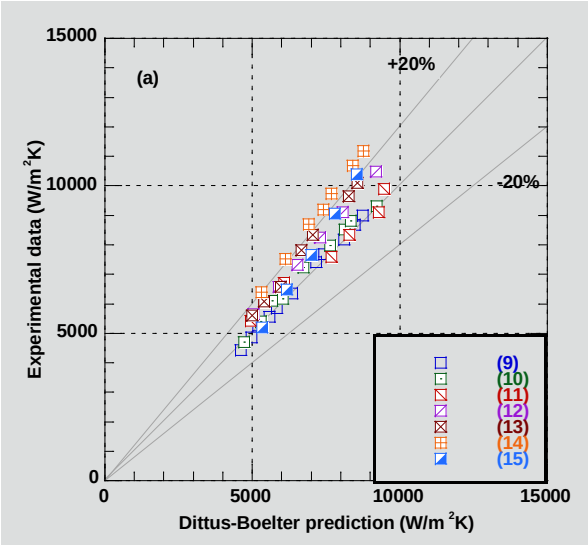


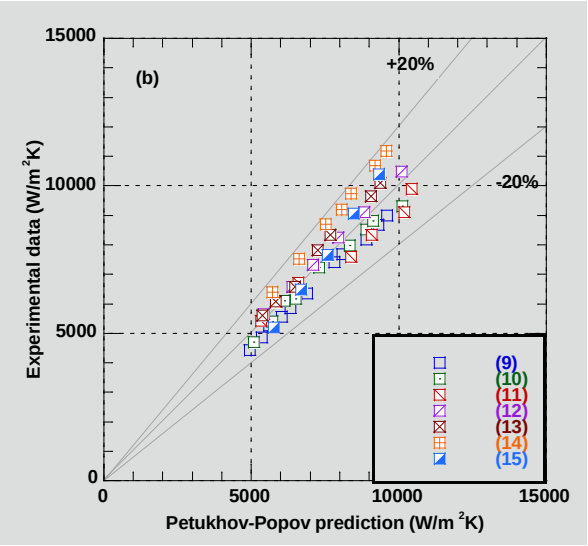


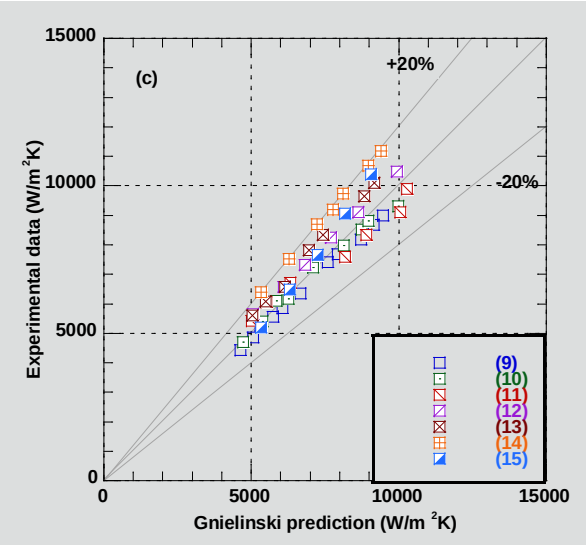


Nanofluid Heat Transfer Coefficient Comparisons: Group B

- Two equations predict most of the experimental data within the ±20% range
  - Copper-in-water nanofluids (Cu in H_2O)

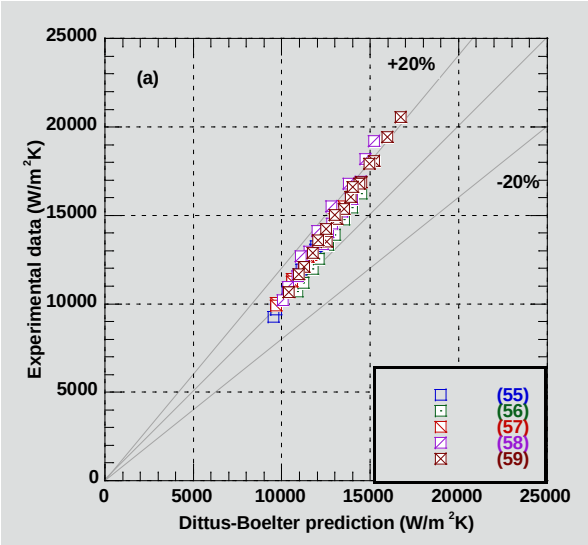


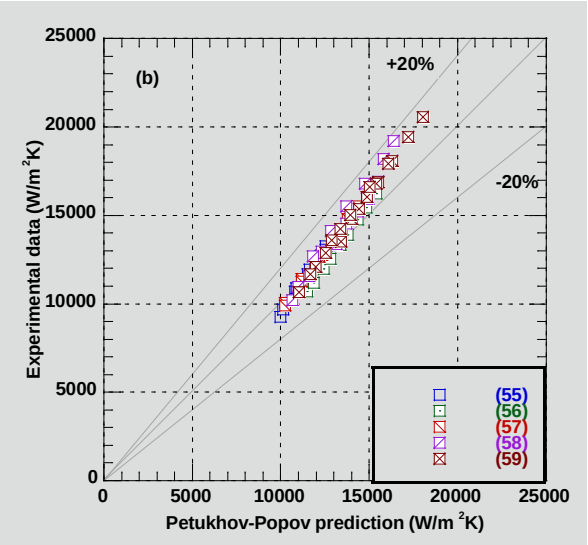


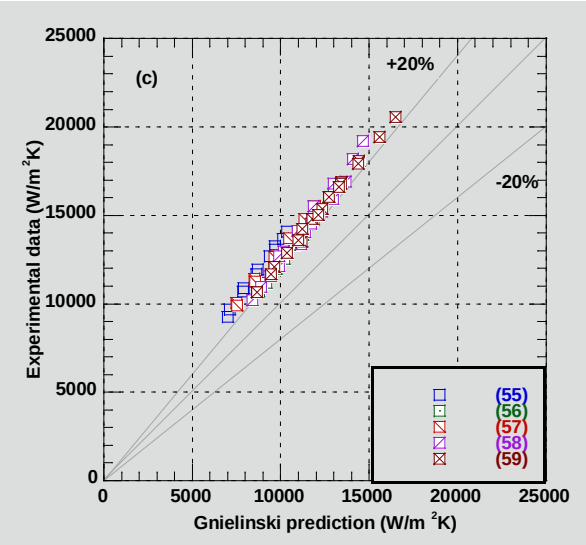


Nanofluid Heat Transfer Coefficient Comparisons: Group B

- Two equations predict most of the experimental data within the ±20% range
  - Silicon carbide-in-50/50 ethylene glycol/water nanofluids (SiC in 50/50 $C_2H_6O_2/H_2O$)

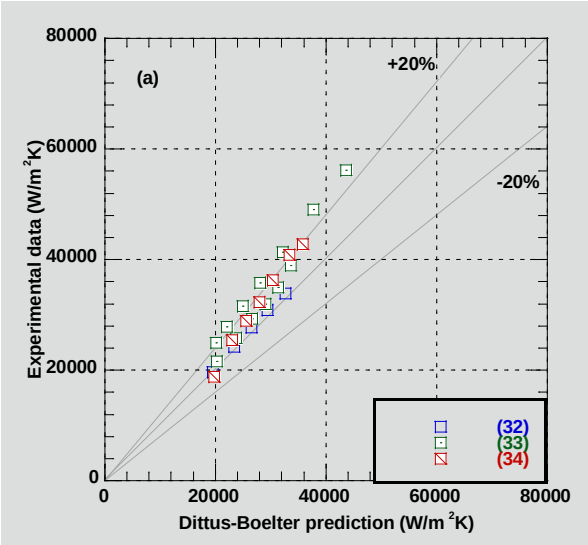


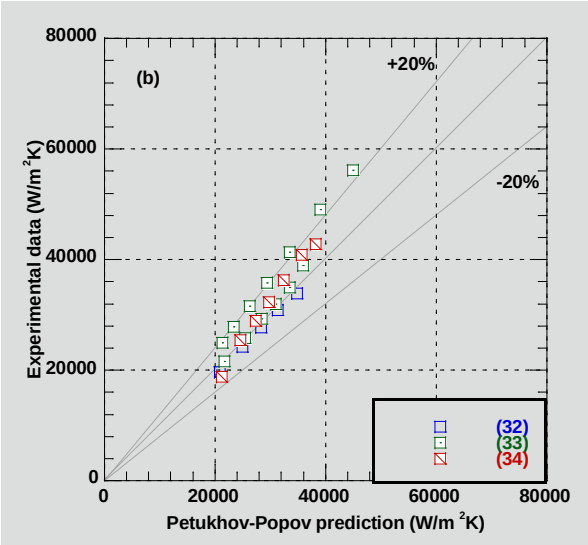


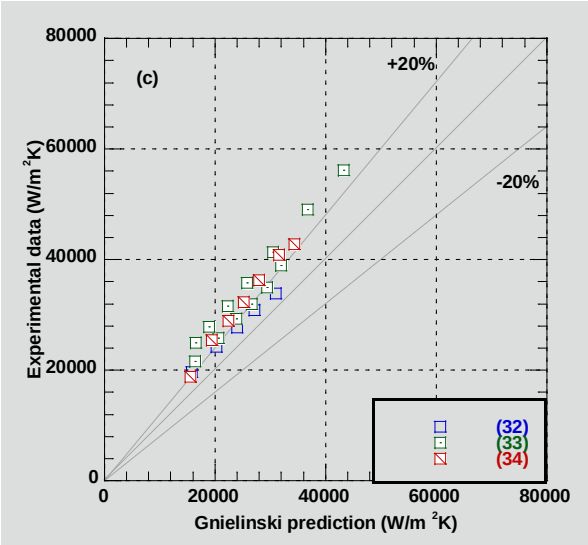


Nanofluid Heat Transfer Coefficient Comparisons: Group C

- One equation predicts most of the experimental data within the ±20% range
  - Multiwalled carbon nanotube-in-water (MWCNT in H_2O)

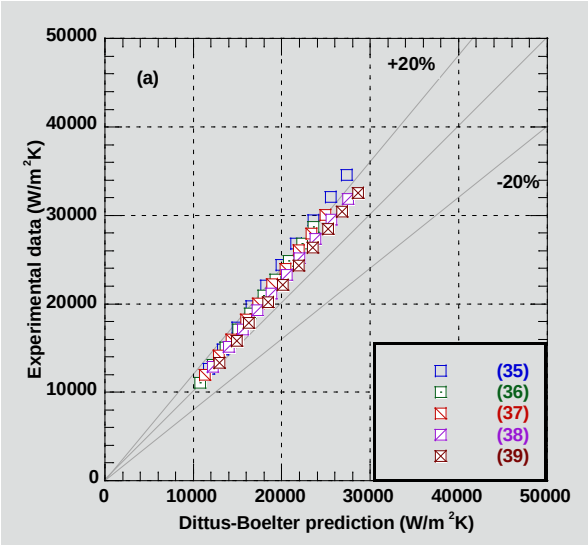


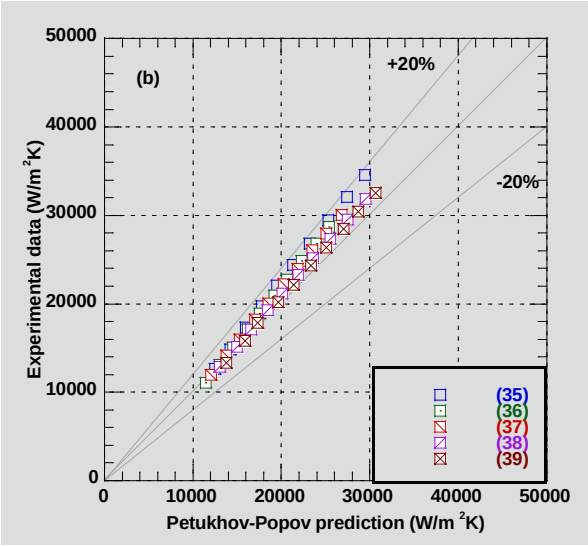


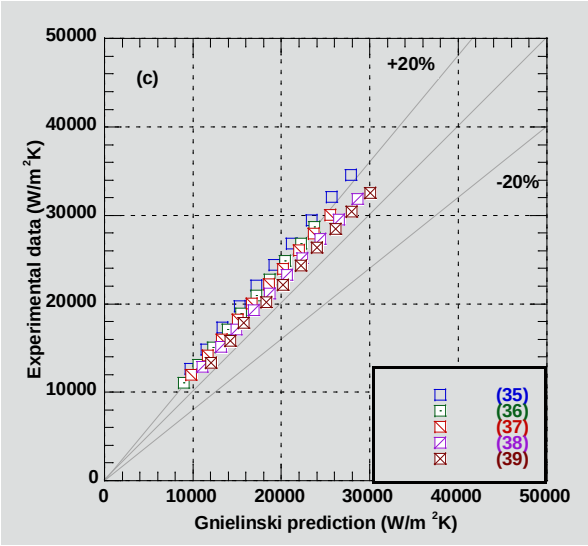


Nanofluid Heat Transfer Coefficient Comparisons: Group C

- One equation predicts most of the experimental data within the ±20% range
  - Silicon carbide-in-water nanofluids (SiC in H_2O)

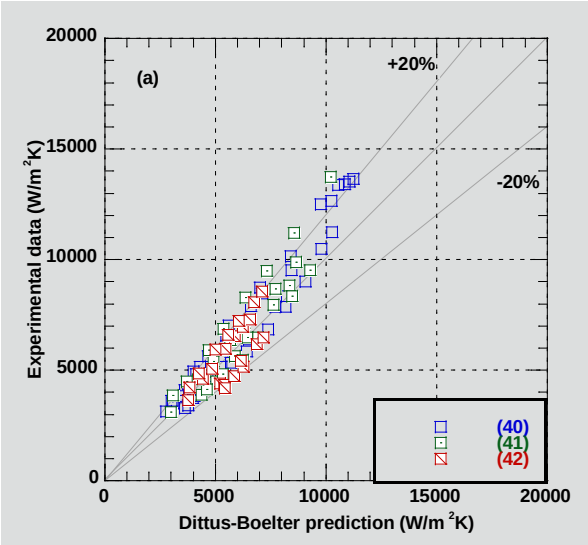


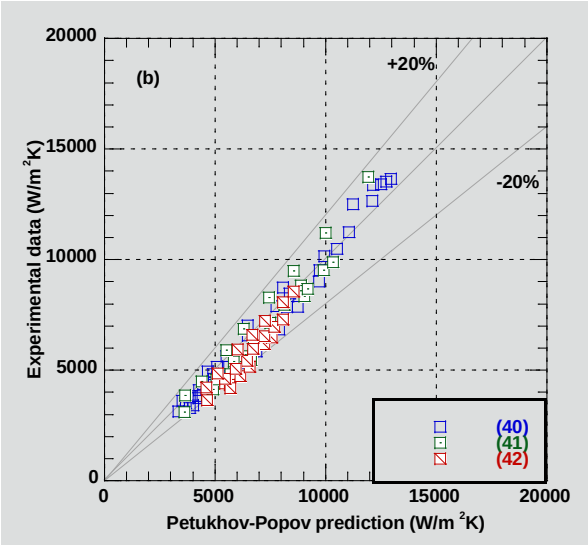


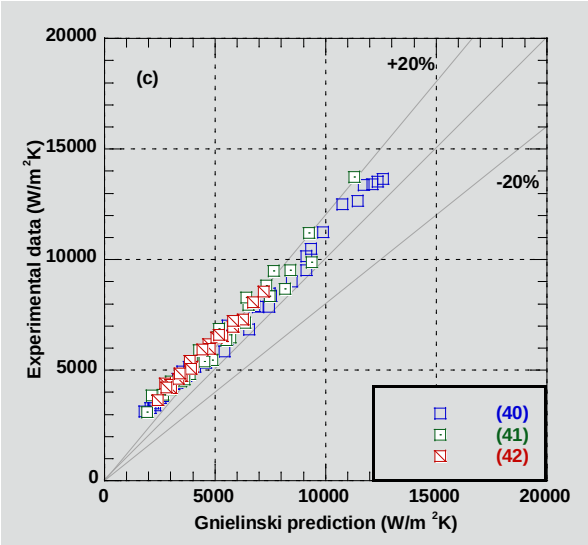


Nanofluid Heat Transfer Coefficient Comparisons: Group C

- One equation predicts most of the experimental data within the ±20% range
  - Silica-in-water nanofluids (SiO_2 in H_2O)

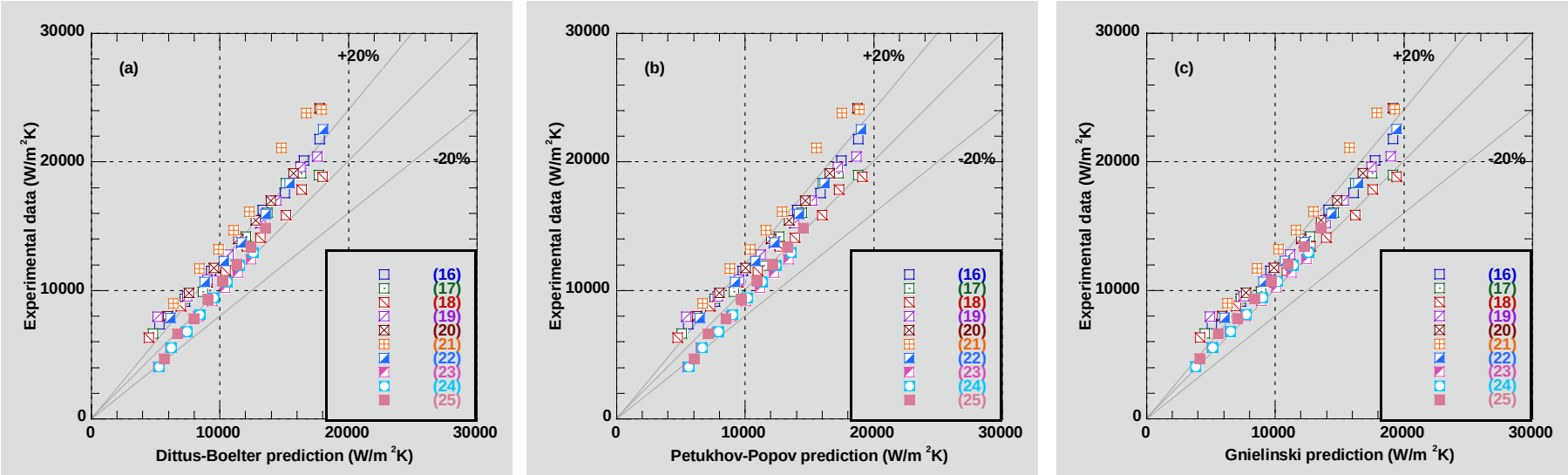






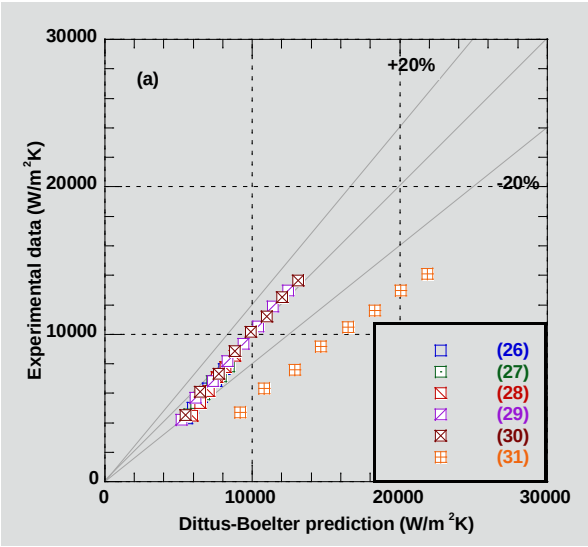
Nanofluid Heat Transfer Coefficient Comparisons: Group D

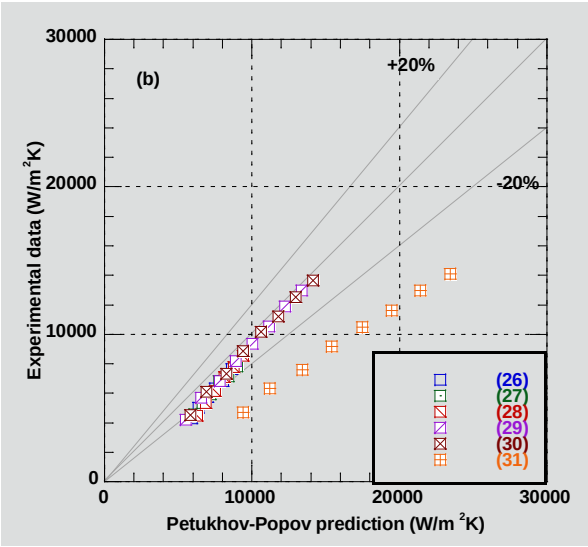
- All three equations predict experimental data trends but with some experimental data outside the ±20% range
  - Cupric oxide-in-water (CuO in H_2O)
  - The predictions are within the ±20% range except
  - Experimental data (16) — (22) (Fotukian and Esfahany 2010)
    - Potential reason: the effective dynamic viscosities and the effective thermal conductivities are not from experimental measurements

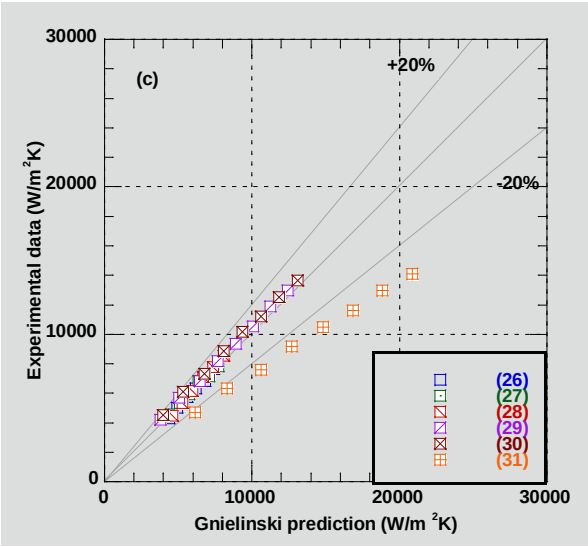


Nanofluid Heat Transfer Coefficient Comparisons: Group D

- All three equations predict experimental data trends but with some experimental data outside the ±20% range
  - Diamond-in-water nanofluids (diamond in H_2O)
  - The predictions are within the ±20% range except
  - Five-volume-percent diamond-in-water experimental data (Torii 2010)
    - Potential reason: the effective dynamic viscosity measurement is considerably higher than and is inconsistent with the rest effective dynamic viscosity measurements







Summary

- For the purpose of heat transfer coefficient predictions, a turbulent-flow nanofluid with a low particle volume concentration can normally be treated as a homogeneous fluid because its particles and base fluid usually are well mixed

- For such a homogeneous nanofluid, the standard single-phase equations will provide quite accurate predictions for its turbulent-flow convective heat transfer coefficients — with the two exceptions, the Petukhov-Popov equation predicts all and every other examined experimental data very well while the Dittus-Boelter equation and the Gnielinski equation predict majority examined experimental data quite well

- The thermophysical property inaccuracy very likely is the reason causing the unsatisfactory predictions — therefore, as far as the effective thermophysical properties of a nanofluid are accurate, the heat transfer coefficient predictions from the standard single-phase equations usually are very good indicators of its actual heat transfer coefficients

Effectiveness of Nanofluids

- The effectiveness of a nanofluid can generally be measured by its heat transfer coefficient enhancements specified usually as the ratio of the heat transfer coefficients of the nanofluid to its base fluid

- The concept of the heat transfer coefficient enhancements of a nanofluid must not be confused with the concept of the heat transfer coefficient predictions of the nanofluid
  - Enhancements — relate the heat transfer coefficients of a nanofluid to those of its base fluid
  - Predictions — relate the heat transfer coefficients of a nanofluid to its effective thermophysical properties
  - Good heat transfer coefficient predictions of a nanofluid imply neither its heat transfer coefficient enhancements nor its heat transfer coefficient deteriorations

- By following the previous approaching, the heat transfer coefficient enhancements are expressed on the comparison bases of the same flow velocity and the same pumping power

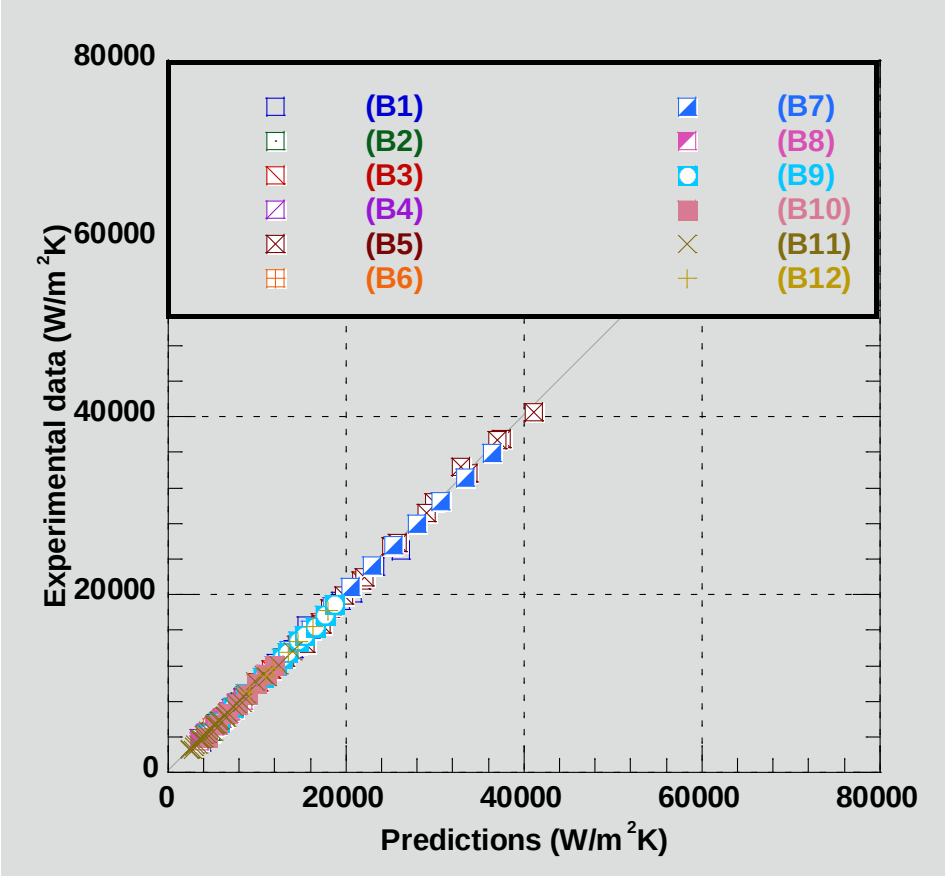
Base Fluid Heat Transfer Coefficients

- The heat transfer coefficients of the base fluid
  - Usually are theoretically predicted
  - Not always follow the same prediction equation even for the same base fluid
- Therefore, different equations or combinations of the standard single-phase equations have been used to acquire very accurate predictions

Author and Year	Base Fluid	Channel (mm)	Prediction Equation
Pak and Cho 1998	(B1) H_2O	10.66	$h_m = h_{Petukhov-Popov}$
Xuan and Li 2003	(B2) H_2O	10	$h_m = h_{Dittus-Boelter}$ for $h_m < 5000$ W/m^{2}K $h_m = (h_{Dittus-Boelter} + h_{Petukhov-Popov})/2$ for $h_m \geq 5000$ W/m^{2}K
Williams et al. 2008	(B3) H_2O	9.398	$h_m = h_{Petukhov-Popov}$
Duangthongsuk and Wongwises 2009	(B4) H_2O	8.13	$h_m = h_{Petukhov-Popov}$ for $h_m < 7000$ W/m^{2}K $h_m = h_{Dittus-Boelter}$ for $h_m \geq 7000$ W/m^{2}K
Liao and Liu 2009	(B5) H_2O	1.02	$h_m = 1.10 h_{Gnielinski}$ for $T = 29$ °C $h_m = h_{Petukhov-Popov}$ for $T = 58$ °C
Torii and Yang 2009	(B6) H_2O	3.96	$h_m = h_{Gnielinski}$
Yu et al. 2009 Timofeeva et al. 2010	(B7) H_2O	2.2733	$h_m = 1.12 h_{Gnielinski}$
Duangthongsuk and Wongwises 2010	(B8) H_2O	8.13	$h_m = 1.13 h_{Dittus-Boelter}$ for $h_m < 7000$ W/m^{2}K $h_m = (h_{Dittus-Boelter} + h_{Petukhov-Popov})/2$ for $h_m \geq 7000$ W/m^{2}K
Fotukian and Esfahany 2010	(B9) H_2O	5	$h_m = h_{Dittus-Boelter}$
Torii 2010	(B10) H_2O	3.96	$h_m = (h_{Dittus-Boelter} + h_{Gnielinski})/2$
Ferrouillat et al. 2011	(B11) H_2O	4	$h_m = (h_{Dittus-Boelter} + h_{Gnielinski})/2$ for heating $h_m = (h_{Dittus-Boelter} + h_{Petukhov-Popov})/2$ for cooling
Timofeeva et al. 2011	(B12) 50/50 $C_2H_6O_2/H_2O$	2.2733	$h_m = 1.28 h_{Gnielinski}$

Base Fluid Heat Transfer Coefficient Predictions

- The predictions are considered to be in excellent agreement with the experimental data



Nanofluid Heat Transfer Coefficients

- The same database for predictions are used for enhancements except
  - Cupric oxide-in-water nanofluids (from Fotukian and Esfahany 2010)
  - Five-volume-percent diamond-in-water nanofluid (from Torii 2010)
- The experimental data are organized according to the component materials

Author and Year	Nanofluid	Concentration Range (%)
Pak and Cho 1998, Williams et al. 2008, Torii 2010	(E1) Al_2O_3 in H_2O	0.1 – 5.0
Xuan and Li 2003	(E2) Cu in H_2O	0.3 – 2.0
Torii 2010	(E3) CuO in H_2O	0.1 – 5.0
Torii and Yang 2009, Torii 2010	(E4) diamond in H_2O	0.1 – 1.0
Liao and Liu 2009	(E5) MWCNT in H_2O	0.243 – 0.978
Yu et al. 2009, Timofeeva et al. 2010	(E6) SiC in H_2O	3.7 – 4.1
Ferrouillat et al. 2011	(E7) SiO_2 in H_2O	2.33 – 18.93
Pak and Cho 1998, Duangthongsuk and Wongwises 2009, Duangthongsuk and Wongwises 2010	(E8) TiO_2 in H_2O	0.20 – 3.16
Williams et al. 2008	(E9) ZrO_2 in H_2O	0.2 – 0.9
Timofeeva et al. 2011	(E10) SiC in 50/50 $C_2H_6O_2/H_2O$	1.0 – 4.0

Heat Transfer Coefficient Enhancement Comparisons

- Comparisons between enhancement ratios and figure-of-merit ratios
  - Enhancement ratios — experimental h_e / predicted h_m under certain flow conditions
  - Figure-of-merit ratios — calculated h_e / calculated h_m from thermophysical properties
- Same flow velocity comparison
  - h_m calculated straightforwardly from the prediction equations with the same flow velocities and temperatures of its corresponding nanofluid
- Same pumping power comparison
  - The pumping power is estimated from the Blasius equation (for the Fanning friction factor)

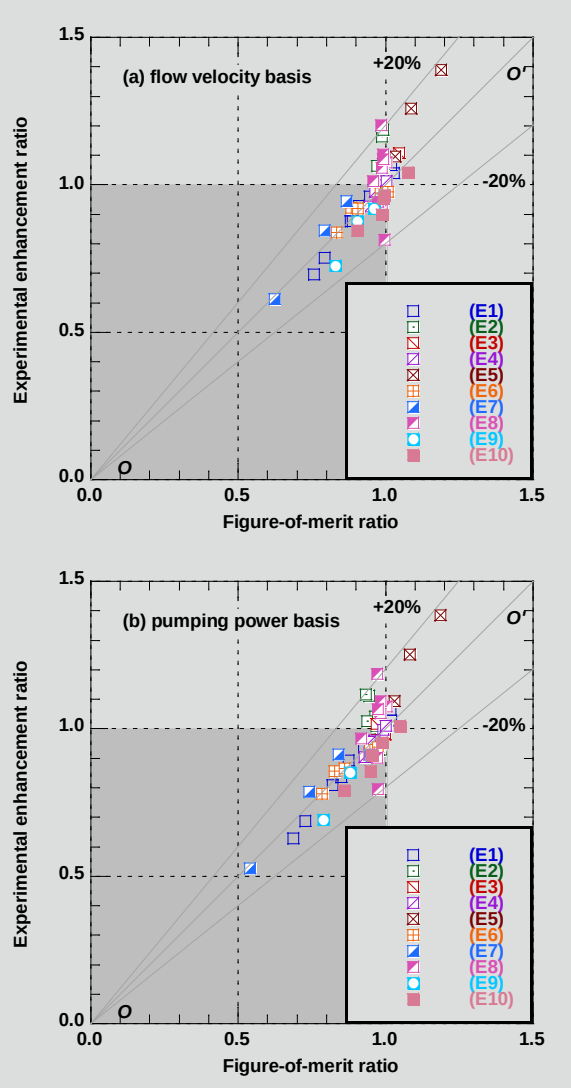
$$P = (\pi/2)dLu^3\rho f = 0.03955\pi d^{3/4}Lu^{11/4}\rho^{3/4}\mu^{1/4}$$

  - The flow velocity for the base fluid is calculated from (P_m=P_e)

$$u_m = (\rho_e/\rho_m)^{3/11}(\mu_e/\mu_m)^{1/11}u_e$$

Overall Heat Transfer Performance — Average Heat Transfer Coefficient Enhancement Comparisons

- The horizontal unity line, the vertical unity line, and the line *OO*'
- Some general trends
  - Experimental heat transfer enhancement ratios agree very with figure-of-merit ratios
  - Figure-of-merit ratios are <1 for experimental heat transfer coefficient enhancement ratios <1 (shaded regime), but not true reversely
  - Almost all (>1) heat transfer coefficient enhancement ratios are greater than the corresponding figure-of-merit ratios
    - Indicating potential additional particle effects on the heat transfer enhancements
  - The same flow velocity comparison and the same pumping power comparison follow the very similar pattern except the enhancement ratios and the figure-of-merit ratios for the latter are smaller, which is expected



Summary

- The heat transfer coefficient enhancements of a nanofluid over its base fluid
  - Dominant influence factor — effective thermophysical properties
  - Potential additional effects — particles

- These results further validate the treatment of a nanofluid as a homogeneous fluid with the effective thermophysical properties

- For such a thermophysical property-dominant homogeneous nanofluid, the actual heat transfer coefficient enhancement ratios under the corresponding comparison bases can be estimated by
  - The figure-of-merit ratios, the heat transfer coefficient ratios of the nanofluid over its base fluid calculated with
  - The Dittus-Boelter equation from
  - The thermophysical properties of the nanofluid and its base fluid for
  - The comparison bases of the same flow velocity and the same pumping power

Conclusions

- Turbulent-flow nanofluids with low particle volume concentrations can normally be treated as homogeneous fluids due to the nature of their well-mixed particles and base fluids
- The additional effects of particles on the convective heat transfer coefficients for such homogeneous nanofluids in turbulent flow are insignificant compared to the dominant influence factor — the effective thermophysical properties of the nanofluids
- The convective heat transfer coefficients of nanofluids in turbulent flow, therefore, can be predicted quite accurately with the standard single-phase equations such as the Dittus-Boelter equation, the Petukhov-Popov equation, or the Gnielinski equation
- The actual heat transfer coefficient enhancement ratios of nanofluids over their base fluids can also be predicted quite accurately with the heat transfer coefficient ratios of the nanofluids over their base fluids calculated with the Dittus-Boelter equation from the thermophysical properties of the nanofluids and their base fluids for the specified comparison basis
- These findings consequently point to the future research directions of engineering nanofluids through optimizing their thermophysical property combinations